

Spherical Relativistic Radiation Flows with Variable Eddington Factor

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Abstract

We solve spherically symmetric radiation flows under full special relativity with the help of a variable Eddington factor $f(\tau, \beta)$, where τ is the optical depth and β is the flow velocity normalized by the speed of light. Relativistic radiation hydrodynamics under the moment formalism has several complex problems, such as a closure relation. Conventional moment equations closed with the traditional Eddington approximation in the comoving frame have singularity, beyond which the flow cannot be accelerated. In order to avoid such a pathological behavior inherent in the relativistic moment formalism, we propose a variable Eddington factor, which depends on the flow velocity as well as the optical depth, for the case of the spherically symmetric one-dimensional flow. We then calculate the relativistic spherical flow with such variable Eddington factors to investigate the case that gas is accelerated by radiative force. As a result, it is shown that the gas speed reaches around the speed of light by radiation pressure.

Key words: astrophysical jets — gamma-ray bursts — radiative transfer — relativity

1. Introduction

Relativistic outflows from a luminous central object are observed in various active phenomena; e.g., relativistic jets and winds from microquasars (Mirabel, Rodríguez 1999; Fender et al. 2004), such as SS 433, GRS 1915+105, GRO J1655–40, jets in active galactic nuclei, such as 3C 273 and gamma-ray bursts (Mészáros 2002). Several mechanisms were proposed to explain these relativistic outflows, including hydrodynamical, radiative, and magnetic drives. When the luminosity highly exceeds the Eddington one, the relativistic outflow seems to be driven by radiation pressure of the central object.

So far relativistic outflows or winds driven by radiation pressure in the spherically symmetric case have been studied by several researchers (Castor 1972; Ruggles, Bath 1979; Mihalas 1980; Quinn, Paczyński 1985; Turolla et al. 1986; Paczyński 1990; King, Pounds 2003). As for numerical calculation, the radiation transfer has been solved in two or three dimensions by a Newtonian treatment, but has not been resolved sufficiently for a highly relativistic case yet.

On the other hand, under the traditional moment formalism, the relativistic outflows driven by radiation pressure have pathological behavior (e.g., Turolla, Nobili 1988; Nobili et al. 1991; Turolla et al. 1995; Dullemond 1999; Fukue 2005). That is to say, moment equations for relativistic radiation transfer can have unphysical critical points. For example, in one-dimensional relativistic radiation flow using the Eddington approximation in the comoving frame, where the moment equations are truncated at the second order, the singularity appears when

the flow velocity becomes $c/\sqrt{3}$. This is understood as follows (Turolla, Nobili 1988; Nobili et al. 1991; Dullemond 1999; Fukue 2006). The radiative diffusion may become *anisotropic* even in the comoving frame of the gas as a result of what the velocity gradient becomes very large in the direction of the flow when the gaseous flow is radiatively accelerated up to the relativistic regime. Hence, in a flow that is accelerated from subrelativistic to relativistic regimes, the Eddington factor should be different from $1/3$ even in the optically thick diffusion limit.

As already stressed in the literature (e.g., Nobili et al. 1991), the location of the critical point in the moment equations depends on the choice of a closure relation, and with a suitable choice of the closure relation, the critical point may disappear. For example, Nobili et al. (1991) adopted a variable Eddington factor, which depends on the optical depth. However, for the present transfer flow, the critical condition, where the denominator of moment equations vanishes, contains the flow velocity (e.g., Nobili et al. 1991; Fukue 2006). Hence, as a natural extension, in the present study we have proposed a variable Eddington factor which depends on the “flow velocity” as well as the optical depth. By adopting such a velocity-dependent Eddington factor, we intend to send the critical point away toward the edge of the speed of light.

In this paper we propose a velocity-dependent Eddington factor $f(\tau, \beta)$ for a spherically symmetric case, and solve the fully special relativistic spherical outflows driven by radiation pressure using such $f(\tau, \beta)$.

In the next section we propose a variable Eddington factor for spherical relativistic radiative flows. In section 3 we describe the basic equations for relativistic outflow driven by radiation pressure under the spherical symmetry. In section 4 we show our numerical results of the radiative flow. The final section is devoted to concluding

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remarks.

2. Variable Eddington Factor

In this section we propose and explain an *optical depth and velocity*-dependent variable Eddington factor, which enables us to treat a problem of relativistic radiation hydrodynamics in a spherically symmetric case.

2.1. Traditional Eddington Factor

We solve the radiation hydrodynamic problem semi-analytically using moment equations. Then, the Eddington approximation is generally used to close moment equations. The usual Eddington approximation is defined in the comoving frame as

$$P_0 = \frac{1}{3}E_0 \quad (1)$$

where P_0 is the radiation pressure and E_0 is the energy density both measured in the comoving frame. This usual Eddington approximation is axiomatic when the radiation field is isotropic. Such a situation can be satisfied, when the atmosphere is sufficiently optically thick, or when the gas is optically thin with the uniform radiation field. As is well-known, however, this usual Eddington approximation does not hold when the radiation field becomes *anisotropic* in such a case that there is a transition from optically thick to thin states.

When we examine the gas accelerated up to the relativistic speed by radiation pressure, as a clue of mechanism for jets in active galactic nuclei and microquasars, it is necessary to consider a sudden change of optical depth and the steep velocity gradient. In order to obtain the terminal speed of the radiatively-driven relativistic outflow, we have to investigate the flow down to the optically thin state. In addition, the usual Eddington approximation would be violated, when the gas is accelerated to the relativistic speed with steep velocity gradient. This is because the radiation field may become anisotropic, even in the comoving frame, due to relativistic aberration and redshift.

2.2. Optical-Depth Dependent Factor

In this subsection, we discuss about a better way of dealing with radiation field which is anisotropic. When there is a transition from optically thick to thin regimes, for a spherically symmetric case Tamazawa et al. (1975) set the Eddington approximation as

$$P_0 = fE_0 \quad (2)$$

where f is a variable Eddington factor, and they proposed the relation that satisfied the physical condition from optically thick to thin regimes by

$$f(\tau) = \frac{1+\tau}{1+3\tau}, \quad (3)$$

where τ is the optical depth.

This factor becomes $1/3$ in an optically thick region while becomes unity in an optically thin region. This is

understood as follows. The photon mean-free path ℓ is on the order of

$$\ell \sim 1/(\kappa\rho) \quad (4)$$

where κ is the opacity and ρ is the gas density. When the gas density is large and the medium is sufficiently thick, the mean free path becomes small and the radiation field is locally seen to be isotropic. While, around the surface of the atmosphere or in a spherically expanding flow, the gas density becomes small and the mean free path lengthens more and more toward the direction of the density gradient; then the radiation field becomes locally seen to be anisotropic. In such a transition region, the relation between the radiation pressure and radiation energy would change in each direction. When the optical depth becomes 0, for an outward direction, the radiation pressure is equal to the radiation energy. However, in the case of relativistic outflow, analytic method can not be calculated until the speed of light with even this factor due to the singularity.

2.3. Velocity Dependent Factor

Next, we consider the case where the gas interacting with photon is accelerated to the relativistic speed. When there is a large velocity gradient, the photon mean-free path becomes longer than that without the velocity gradient. In such a case, the usual Eddington approximation would be violated again. For instant, in the relativistic flow with a velocity gradient dv/dr , where v is the flow velocity and r the radius, the velocity increase at a distance of the mean free path ℓ becomes

$$\Delta v = \ell \frac{dv}{dr} = \frac{1}{\kappa\rho} \frac{dv}{dr} \sim \frac{dv}{d\tau}. \quad (5)$$

In order for the radiation fields to be isotropic in the comoving frame, this velocity increase should be sufficiently smaller than the speed of light; $dv/d\tau \sim v/\tau \ll c$. If the velocity difference becomes very large when the velocity itself is very high and/or the optical depth is small, the usual Eddington approximation in the comoving frame would be violated. Such a situation can occur for a relativistic outflow. If the velocity difference is large at a distance of the mean free path, the relativistic effect, such as a Doppler effect and aberration, becomes important, and the radiation field is seen to be anisotropic.

For a relativistic flow with a velocity gradient, a velocity-dependent variable Eddington factor was proposed (Fukue 2006):

$$f(\beta) = \frac{1}{3} + \frac{2}{3}\beta, \quad (6)$$

where $\beta = v/c$. This factor is applied to the plane-parallel case. As a good news by the usage of this velocity-dependent factor, we can avoid critical points that always appear in the moment equations under special relativity. In this point, it is indicated that the velocity-dependent variable Eddington factor in the relativistic flow could be reasonable mathematically as well as physically. However, in the spherical case we also have to consider about the effect of the optical depth against to the plane-parallel

case which does not include the effect of the optical depth through velocity of gas.

2.4. Optical-Depth and Velocity Dependent Factor

Now, we consider the case of a relativistic spherical flow. The Eddington factor depends on the optical depth for a spherical atmosphere, while it depends on the flow velocity for a relativistic flow. In the spherically symmetric relativistic flow, there exist a dilution effect due to a spherical expansion and that due to a relativistic expansion. Hence, we suppose that the Eddington factor could depend on both the optical depth *and* the flow velocity. The minimum requirements for such a variable Eddington factor are (i) it approaches $1/3$ in a sufficiently thick, low velocity regime, (ii) it becomes unity in an optically thin regime, and (iii) it does also become unity in the relativistic regime at a speed on the order of the speed of light. Additional conditions are (iv) it reduces to the factor of Tamazawa et al. (1975) in a static limit, and (v) it is simple.

Although there may be many possible factors, in the present paper we propose the following one,

$$f(\tau, \beta) = \frac{\gamma(1 + \beta) + \tau}{\gamma(1 + \beta) + 3\tau}, \quad (7)$$

where τ is the optical depth, β is the normalized flow speed ($\beta = v/c$), and γ is the Lorentz factor [$\gamma = 1/\sqrt{1 - (v/c)^2}$]. This form was born as follows. It is shown that the mean free path ℓ of photons in the inertial frame lengthens than that ℓ_0 in the comoving frame by a relativistic effect (Abramowicz et al. 1991) as $\ell = \ell_0/[\gamma(1 - \beta \cos \theta)] = \ell_0 \gamma(1 + \beta)$. By considering this, we replaced the optical depth of Tamazawa et al (1975) by $\tau/[\gamma(1 + \beta)]$ for the outward moving flow.

Figure 1 shows the behavior of the present variable Eddington factor (7). A dashed curve is the variable Eddington factor by Tamazawa et al. (1975), while other curves are the present case for several values of the flow speed. As is seen in figure 1, the present variable Eddington factor becomes unity as the flow speed approaches the speed of light.

Using these variable Eddington factors, we can calculate the spherically symmetric relativistic flow, continuously from low speed to relativistic regimes. In the next section, we solve the relativistic moment equations with a variable Eddington factor for the relativistic spherically symmetric case.

3. Basic Equations

In this paper, it is treated a simple one-dimensional radiation flow in what follows; i.e., we consider the spherical case in the radial direction. The radiative energy is transported in the radial direction, and the gas itself also moves in the radial direction by the action of radiation pressure. For simplicity, the radiation field is sufficiently intense that both the gravitational field, e.g., of the central object, and the gas pressure and the internal heating are ignored in this paper. As for the order of the flow ve-

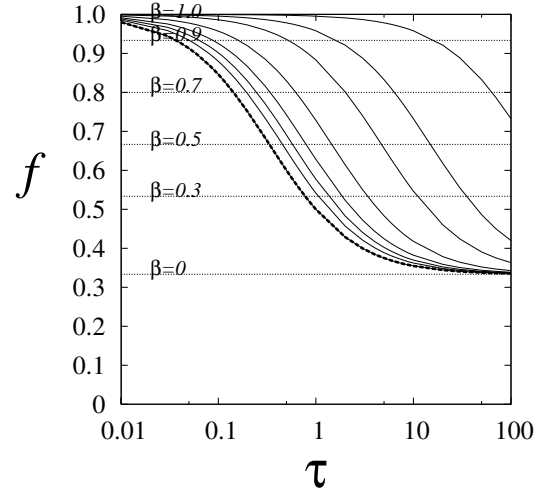


Fig. 1. Optical depth and velocity dependent variable Eddington factor. The dashed curve shows an optical depth dependent variable Eddington factor, $f(\tau) = (1 + \tau)/(1 + 3\tau)$. The dotted lines denote the velocity dependent variable Eddington factor, $f(\beta) = 1/3 + (2/3)\beta$, where the velocity is $\beta = 0.3, 0.5, 0.7, 0.9, 0.99, 0.999, 0.99999$ from bottom to top. The solid curves represent an optical depth and velocity dependent variable Eddington factor, $f(\tau, \beta) = \{\gamma(1 + \beta) + \tau\}/\{\gamma(1 + \beta) + 3\tau\}$, where the velocity is $\beta = 0.3, 0.5, 0.7, 0.9, 0.99, 0.999, 0.99999$ from bottom-left to top-right.

locity v , we consider the fully special relativistic regime, where the all terms are retained. Importance to retain the higher order of velocity is described in Yin and Miller (1995). Under these assumptions, the radiation hydrodynamic equations for steady radial (r) flows are described as follows (Kato et al. 1998; cf. Fukue 2006 for a plane-parallel case).

The continuity equation is

$$4\pi r^2 \rho c u = \dot{M} \quad (= \text{const.}), \quad (8)$$

where ρ is the proper gas density, u the radial four velocity, \dot{M} the mass-loss rate, and c the speed of light. The four velocity u is related to the proper three velocity v by $u = \gamma v/c$.

The equation of motion is

$$c^2 u \frac{du}{dr} = \frac{\kappa_{\text{abs}} + \kappa_{\text{sca}}}{c} [F\gamma(1 + 2u^2) - c(E + P)\gamma^2 u], \quad (9)$$

where κ_{abs} and κ_{sca} are the absorption and scattering opacities (gray), which relate to bremsstrahlung, photoionization and electron scattering. We define them in the comoving frame. Moreover, E is the radiation energy density, F the radiative flux, and P the radiation pressure observed in the inertial frame. In the no-gas pressure approximation and without heating, the energy equation is reduced to a radiative equilibrium relation,

$$0 = j - c\kappa_{\text{abs}} E \gamma^2 - c\kappa_{\text{abs}} P u^2 + 2\kappa_{\text{abs}} F \gamma u, \quad (10)$$

where j is the emissivity defined in the comoving frame. In this equation (10), the third and fourth terms on the

right-hand side appear in the relativistic regime.

For radiation fields, the zeroth-moment equation becomes

$$\frac{1}{r^2} \frac{d}{dr} (r^2 F) = \rho \gamma [j - c\kappa_{\text{abs}} E + c\kappa_{\text{sca}} (E + P) u^2 + \kappa_{\text{abs}} F u / \gamma - \kappa_{\text{sca}} F (1 + v^2/c^2) \gamma u]. \quad (11)$$

The first-moment equation is

$$\frac{dP}{dr} = -\frac{1}{r} (3P - E) + \frac{\rho \gamma}{c} [ju/\gamma - \kappa_{\text{abs}} F + c\kappa_{\text{abs}} Pu/\gamma - \kappa_{\text{sca}} F (1 + 2u^2) + c\kappa_{\text{sca}} (E + P) \gamma u]. \quad (12)$$

Although the first term on the right-hand side of equation (12) disappears for the closure relation such as an Eddington approximation in the optically thick limit, this term remains in the equation because of the modification of the Eddington approximation in this paper. It should be noted that this equation (12) is reduced to that by Ruggles and Bath (1979) in the lower approximation of $(v/c)^1$.

Here, in order to close moment equations for radiation fields, we adopt a velocity-dependent or optical depth and velocity-dependent variable Eddington approximation (2). If we adopt this form (2) as the closure relation in the comoving frame, the transformed closure relation in the inertial frame is

$$cP(1 + u^2 - fu^2) = cE(f\gamma^2 - u^2) + 2F\gamma u(1 - f), \quad (13)$$

or equivalently,

$$cP(1 - f\beta^2) = cE(f - \beta^2) + 2F\beta(1 - f). \quad (14)$$

Above closure relation gives the relation among radiation pressure, energy and flux. Relations among E , F , and P , which depend on velocity, are important relations, and it is a point on using the modified closure relation.

Eliminating j with the help of equations (10) and using continuity equation (8), equations (9), (11) and (12) are rearranged as

$$c\dot{M} \frac{du}{dr} = 4\pi r^2 \rho \frac{\gamma}{c} (\kappa_{\text{abs}} + \kappa_{\text{sca}}) \times [F(1 + 2u^2) - c(E + P)\gamma u], \quad (15)$$

$$\frac{d}{dr} (r^2 F) = r^2 \rho u (\kappa_{\text{abs}} + \kappa_{\text{sca}}) \times [c(E + P)\gamma u - F(1 + 2u^2)], \quad (16)$$

$$\frac{dP}{dr} = -\frac{1}{r} (3P - E) + (\kappa_{\text{abs}} + \kappa_{\text{sca}}) \times \rho \frac{\gamma}{c} [c(E + P)\gamma u - F(1 + 2u^2)]. \quad (17)$$

The integration of the sum of equations (15) and (16) yields the energy flux conservation along the flow,

$$c^2 \dot{M} \gamma + L = c^2 \dot{M} + L_0 \quad (= \text{const.}), \quad (18)$$

where $L (= 4\pi r^2 F)$ is the luminosity. The initial conditions are given as $u = 0$, $L = L_0$, $P = P_0$, and $r = r_0$ at $\tau = \tau_0$. The subscript zero denotes the values at the flow base of $\tau = \tau_0$. On the basis of above the basic equations are the equation of motion (15), the mass flux (8), the

momentum flux (17), the energy flux (18) and the closure relation (14) at this stage.

Here, we define new variables for convenient calculations: $Q = 4\pi r^2 cP$ for radiation pressure and $D = 4\pi r^2 cE$ for radiation energy. Substituting these variables into equations (15), (17), and (18), with the help of equation (14), we obtain

$$c^2 \dot{M} \gamma^3 \frac{d\beta}{dr} = (\kappa_{\text{abs}} + \kappa_{\text{sca}}) \rho \gamma \times \frac{(f + \beta^2)L - \beta Q(1 + f)}{f - \beta^2}, \quad (19)$$

$$\frac{dQ}{dr} = -(\kappa_{\text{abs}} + \kappa_{\text{sca}}) \rho \gamma \frac{(f + \beta^2)L - \beta Q(1 + f)}{f - \beta^2} + \frac{1}{r} \frac{(1 - f)(1 + \beta^2)Q - 2\beta L(1 - f)}{f - \beta^2}, \quad (20)$$

$$\dot{M} c^2 \gamma + L = \dot{M} c^2 + L_0. \quad (21)$$

In addition, we regard the optical depth τ as

$$d\tau = -(\kappa_{\text{abs}} + \kappa_{\text{sca}}) \rho dr, \quad (22)$$

and the mass flux (8), the momentum (19), the first moment (20), and the energy flux (21) are rewritten as

$$\frac{dr}{d\tau} = -\frac{4\pi r^2 c \gamma \beta}{(\kappa_{\text{abs}} + \kappa_{\text{sca}}) \dot{M}}, \quad (23)$$

$$c^2 \dot{M} \gamma^3 \frac{d\beta}{d\tau} = -\gamma \frac{(\beta^2 + f)L - (1 + f)\beta Q}{f - \beta^2}, \quad (24)$$

$$\frac{dQ}{d\tau} = -\frac{4\pi r c \gamma \beta}{(\kappa_{\text{abs}} + \kappa_{\text{sca}}) \dot{M}} \frac{(1 - f)[(1 + \beta^2)Q - 2\beta L]}{f - \beta^2} + \gamma \frac{(f + \beta^2)L - \beta Q(1 + f)}{f - \beta^2}, \quad (25)$$

$$\dot{M} c^2 \gamma + L = \dot{M} c^2 + L_0. \quad (26)$$

In order to transform them into dimensionless forms, the radius r is normalized by the Schwarzschild radius $r_g (= 2GM/c^2)$, the mass-loss rate \dot{M} is normalized by L_E/c^2 , and the pressure Q and luminosity L are normalized by the Eddington luminosity $L_E [= 4\pi cGM/(\kappa_{\text{abs}} + \kappa_{\text{sca}})]$. They can be rewritten as

$$\frac{d\hat{r}}{d\tau} = -\frac{2\hat{r}^2 \gamma \beta}{\hat{M}}, \quad (27)$$

$$\hat{M} \gamma^3 \frac{d\beta}{d\tau} = -\gamma \frac{(\beta^2 + f)\hat{L} - (1 + f)\beta \hat{Q}}{f - \beta^2}, \quad (28)$$

$$\frac{d\hat{Q}}{d\tau} = -\frac{2\hat{r} \gamma \beta}{\hat{M}} \frac{(1 - f)[(1 + \beta^2)\hat{Q} - 2\beta \hat{L}]}{f - \beta^2} + \gamma \frac{(f + \beta^2)\hat{L} - \beta \hat{Q}(1 + f)}{f - \beta^2}, \quad (29)$$

$$\hat{M} \gamma + \hat{L} = \hat{M} + \hat{L}_0. \quad (30)$$

At this “first” normalization stage, we briefly comment the boundary conditions on the present case. Moment equations are to be solved as a two-point boundary value problem, as is well known. That is, at the base, flow deep inside the atmosphere, several conditions are imposed on

the physical quantities for radiation fields, whereas, at the surface of the atmosphere, some relation generally holds on the radiative moments with or without the external irradiation. In the present radiative flow, we give the boundary conditions r_0 , $\beta(=0)$, Q_0 (or P_0), and L_0 at the flow base of the optical depth τ_0 . In addition, there exists some relation for the moment Q and L (Fukue 2006) at the flow top of the optical depth of $\tau = 0$. Then, the mass-loss rate \dot{M} should be determined as an eigen value by the boundary condition at the flow top. Although it indeed be possible we do the “second” normalization below.

In the present treatment, we only consider the radiation field without gravitational field add up to nothing characteristic scale expected for mass-loss rate; i.e. mass-loss rate itself can be absorbed in the normalization unit. We further renormalize the variables by $\tilde{r} = \hat{r}/\hat{M}$, $\tilde{L} = \hat{L}/\hat{M}$, $\tilde{Q} = \hat{Q}/\hat{M}$ to yield

$$\frac{d\tilde{r}}{d\tau} = -2\tilde{r}^2\gamma\beta, \quad (31)$$

$$\gamma^3 \frac{d\beta}{d\tau} = -\gamma \frac{(\beta^2 + f)\tilde{L} - (1 + f)\beta\tilde{Q}}{f - \beta^2}, \quad (32)$$

$$\begin{aligned} \frac{d\tilde{Q}}{d\tau} = & -2\tilde{r}\gamma\beta \frac{(1 - f)[(1 + \beta^2)\tilde{Q} - 2\beta\tilde{L}]}{f - \beta^2} \\ & + \gamma \frac{(f + \beta^2)\tilde{L} - \beta\tilde{Q}(1 + f)}{f - \beta^2}, \end{aligned} \quad (33)$$

$$\gamma + \tilde{L} = 1 + \tilde{L}_0. \quad (34)$$

At this “second” normalization stage, the mass-loss rate apparently disappears in the basic equations and it seems unnecessary the boundary condition at the flow top to determine the mass-loss rate. Thus, we solve equations (31)–(34) for a suitable form of variable Eddington factors $f(\tau, \beta)$.

4. Results and Discussion

In this section we briefly show a typical example for the relativistic spherical flow using the present variable Eddington factor, and discuss and compare several forms of variable Eddington factors.

4.1. Typical Example with Fastest Terminal Velocity

We first show a typical example of the relativistic spherical flow, after solving the special relativistic radiation hydrodynamic equations, using the present proposed factor (7).

Among various combinations of parameters, we find the case of fastest terminal velocity for the initial condition at the flow base: $\tilde{L}_0 = 1$, $\tilde{Q}_0 = 0.99$, $\tilde{r}_0 = 1$, $\beta_0 = 0$, and $\tau_0 = 1$. In this case, the terminal speed becomes $0.68c$. The result is shown in figure 2. As is seen in figure 2, the gas is accelerated as the luminosity decreases; i.e., the radiation energy is converted to the bulk motion in such a relativistic regime. The gas is accelerated at around the flow top of $\tilde{r} \sim 5[r_g c^2/L_E]$, where the optical depth vanishes. It is

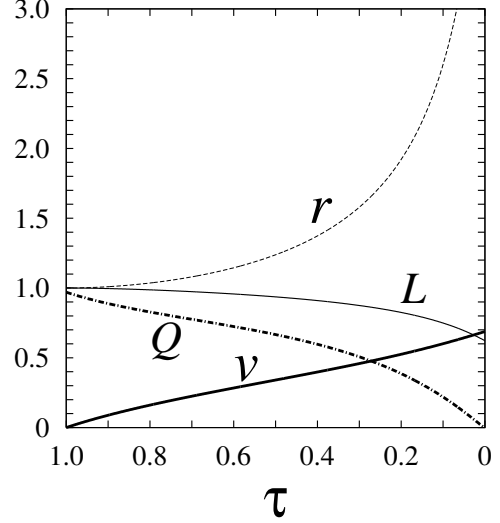


Fig. 2. Flow three velocity v (thick solid curve), radiative luminosity L (thin solid curve), radiation pressure Q (chain-dotted curve), and radial distance r (dotted curve) as a function of the optical depth τ for $\tilde{r}_0 = 1$ at the flow base of $\tau_0 = 1$. Other parameters are $\tilde{L}_0 = 1$ and $\tilde{Q}_0 = 0.99$.

stressed that there does not appear pathological critical points inherent in the usual Eddington factor of $1/3$.

Although we can find the relativistic flow beyond the critical points, it is difficult to obtain the solutions with terminal velocities of $\sim c$. There are several reasons.

The first is the restriction from energy conservation (34). Because the total energy is constant, even if all of the radiation energy is converted to the bulk energy, the terminal Lorentz factor γ_∞ is restricted as

$$\gamma_\infty \leq 1 + \tilde{L}_0. \quad (35)$$

In the case of figure 2, the terminal speed is smaller than this absolute limit.

The second reason is the existence of radiation drag. On the right-hand side of equation (32), the term of \tilde{L} is the radiative acceleration, while the term related to \tilde{Q} means the radiation drag force, which is approximately proportional to the flow speed. The terminal speed generally becomes high for large luminosities. At the same time, however, the radiation drag force becomes important as the flow speed is high. In the optically thin regime, radiation drag becomes important.

The third is the dilution (curvature) effect of the spherical flow, which does not exist in the plane-parallel case (Fukue 2006). For a simple discussion, we assume that the flow speed is constant with the terminal value of β_∞ and γ_∞ . In such a case, the continuity equation (31) is integrated from the flow base of r_0 to top of r_∞ as

$$\frac{1}{r_0} - \frac{1}{r_\infty} = 2\gamma_\infty\beta_\infty\tau_0. \quad (36)$$

Or, there is a restriction of

$$\gamma_{\infty}\beta_{\infty} < \frac{1}{2r_0\tau_0} \quad (37)$$

for finite r_0 . The terminal speed in figure 2 is on the order of this dilution (curvature) limit.

4.2. Comparison with Various Factors

In this subsection we compare the results for various variable Eddington factors.

In figures 3–5 the velocity, the luminosity, and the radius are shown respectively as a function of the optical depth for parameters of $L_0 = 1$, $Q_0 = 1$, and $r_0 = 1$. The thin solid curve is results for usual Eddington factor which is constant. The chain-dotted curve is the one for the factor of Tamazawa et al.(1975) which depends on optical depth. The dashed curve is the one for the factor is Fukue(2006) which depends on velocity. The thick solid curve is the one for the factor of proposed factor in this paper which depends on velocity and optical depth.

As seen in figure 3, the result for usual Eddington factor shows an acceleration of the flow is not enough due to the singular or radiation drag. On the other hand, in the case of the other factors it can be also shown that the terminal velocity of gas becomes large, even for the same parameters. In this case there is a difference of around 15% in each factor. Although it seems to close these results for Tamazawa’s factor and present proposed one, the latter is physically acceptable as discussed in section 2.

As seen in figure 4, the luminosity is converted to the bulk motion. That is, the luminosity decrease for the usual factor is small, whereas that for other factors is large up to 10% ~ 15%. Such a luminosity change may be a clue to discriminate the various Eddington factors.

Finally, figure 5 shows the radius change of the expanding photosphere. The terminal speed is large as the accelerating distance becomes large. In the present example, the radius is at most $\tilde{r} \sim 4[r_g c^2/L_E]$. For large optical depth, this radius would be large, and the terminal speed would also become large. These results show that gas accelerate as a stretch in the vicinity of the center of compact objects.

5. Concluding Remarks

In the present paper, we examine the relativistic radiation flow in the spherically symmetric case with the velocity- and optical depth- dependent variable Eddington factors within the framework of special relativity. We showed that in the relativistic spherical flow the Eddington factor is no longer constant, but depends on the velocity as well as the optical depth. In particular, when the gas is accelerated up to the relativistic speed, there exists a strong velocity gradient, and the velocity dependence of the Eddington factor becomes important. In addition, such a variable factor can avoid the pathological singularity in the moment equations. We emphasize that we should use such a generalized Eddington factor to treat the relativistic radiation hydrodynamics under the moment formalism.

We can find several solutions for the relativistic spheri-

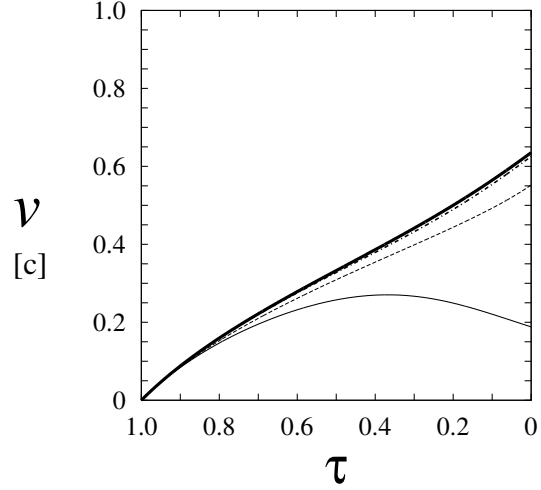


Fig. 3. Velocity as a function of the optical depth for parameters of $L_0 = 1$, $Q_0 = 1$, and $r_0 = 1$. The thin solid curve is for the case of $f = 1/3$, the chain-dotted one for $f(\tau) = (1 + \tau)/(1 + 3\tau)$, the dashed one for $f(\beta) = (1 + 2\beta)/3$, the thick solid one for $f(\tau, \beta) = \{\gamma(1 + \beta) + \tau\}/\{\gamma(1 + \beta) + 3\tau\}$.

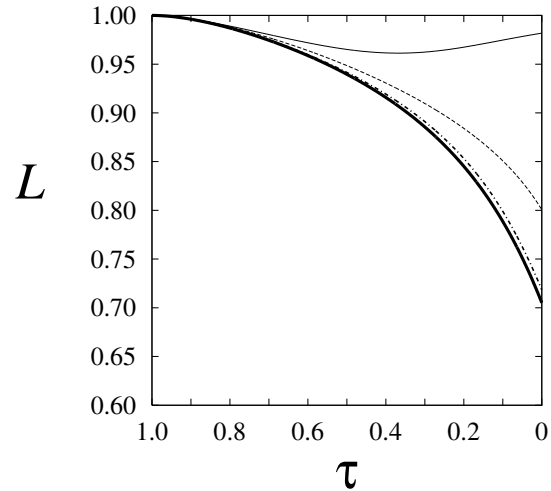


Fig. 4. Luminosity as a function of the optical depth for parameters of $L_0 = 1$, $Q_0 = 1$, and $r_0 = 1$. The thin solid curve is for the case of $f = 1/3$, the chain-dotted one for $f(\tau) = (1 + \tau)/(1 + 3\tau)$, the dashed one for $f(\beta) = (1 + 2\beta)/3$, the thick solid one for $f(\tau, \beta) = \{\gamma(1 + \beta) + \tau\}/\{\gamma(1 + \beta) + 3\tau\}$.

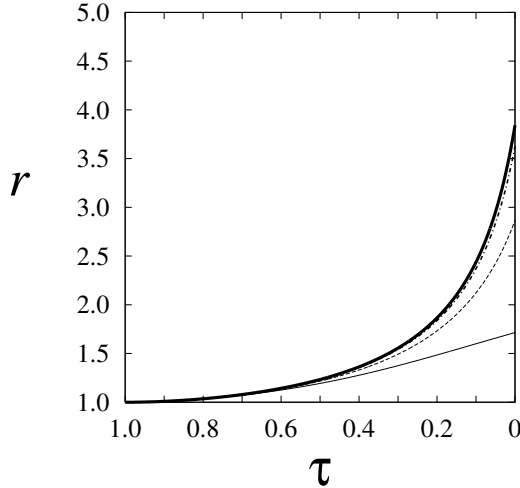


Fig. 5. Radius as a function of the optical depth for parameters of $L_0 = 1$, $Q_0 = 1$, and $r_0 = 1$. The thin solid curve is for the case of $f = 1/3$, the chain-dotted one for $f(\tau) = (1 + \tau)/(1 + 3\tau)$, the dashed one for $f(\beta) = (1 + 2\beta)/3$, the thick solid one for $f(\tau, \beta) = \{\gamma(1 + \beta) + \tau\}/\{\gamma(1 + \beta) + 3\tau\}$.

cal flow. The results, however, are slightly different for the Eddington factor adopted. In order to determine the precise form of the Eddington factor, we must solve the relativistic transfer equation rigorously. However, the functional form of the variable Eddington factor is useful for the study of the relativistic jets, black-hole winds, and the gamma-ray bursts.

It should be commented on the current works on the related topics. Current works are divided mainly into two categories, as referred in the introduction. One type solved the relativistic radiation hydrodynamical equations under the diffusion approximation (e.g., Ruggles, Bath 1979; Quinn, Paczyński 1985; Paczyński, Prószyński 1986; Turolla et al. 1986; Paczyński 1990; Nobili et al. 1994). In these current works the flow is restricted in the sub-relativistic region on the order of $\sim 0.1 c$. However, the diffusion approximation may be valid only in the sufficiently optically thick regime, and further, there is no justification that the diffusion approximation can be used in the relativistic regime, since there exists a causality problem. Another type examined the pathological behavior of the traditional moment formalism in the relativistic regime (e.g., Turolla, Nobili 1988; Nobili et al. 1991; Turolla et al. 1995; Dullemond 1999; Fukue 2005), which is one of the motivation of the present study. However, there is no proposal to use a variable Eddington factor, which depends on the flow velocity as well as the optical depth, in order to solve the moment equations in the relativistic regime in the spherically symmetric case. We thus tried to solve the relativistic moment equations with an approximate form of the variable Eddington factor.

In this paper, we considered only the one-dimensional case without gravity under special relativity. In order to clarify the physics of relativistic jets around a black hole,

we must treat the problem within the framework of general relativity. Such a case is a next work.

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